WHAT WE UNDERSTAND TODAY ON FORMANTS IN SAXOPHONE SOUNDS?

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ABSTRACT

The question of the formants in saxophone sounds involves several paradoxes. The analogy with "cylindrical saxophones", i.e. cylindrical tubes excited at a given proportion of the length, is classical and can be extended to the bowed string. This analogy leads to an approximation of the spectrum of the pressure inside the mouthpiece valid only at low frequencies. Nevertheless it gives good results even at higher frequency, this paradox being now understood. The spectrum of the external pressure contains formants which are different from that of the internal spectrum. The question of what is the cause of the formants remains open.

1. Introduction

A formant (resp. an anti-formant) can be defined as a reinforced (resp. attenuated) frequency band whatever the played note. It is usually assumed that it is an important characteristic of the tone colour, especially for conical reed instruments. Formants need to be distinguished from other timbre characteristics, such as the weakness of harmonics of a given rank (e.g. the even harmonics in the clarinet sound). If formants (or anti-formants) exist, by definition their frequencies cannot depend on the total equivalent length of the tube for a given note, but either on other geometrical parameters (input radius, apex angle of the truncated cone, dimensions of the mouthpiece) or excitation parameters. The present paper is devoted to the study of the dependence of formant frequencies to the geometrical parameters.

The statement of the problem is ancient. As examples we can cite the works by Smith and Mercer (1974) or Benade (1980), who wrote: *I should comment here that much of the formant structure traditionally attributed to formant spectra (to the extent that the measurements are correct at all) is in fact due to the rise and fall of the spectrum envelope produced by the beating reed. We recall that the strict usage of the word formant refers to the enhancement of certain portions of a sound spectrum that is associated with more or less invariable resonance or radiation maxima in the air* *column. It is worthwhile to look forward here to one of the conclusions we will reach in the course of this lecture. There is in fact almost no simple formant behaviour to be recognized in the sound production of wind instruments*. Some studies on the psycho-acoustical aspects can be found in Gridley (1987) and Nykänen et al (2009). The existence of characteristic frequencies in the spectrum of conical reed instruments is closely related to the existence of characteristic times. For bassoon sounds, Gokhstein (1979) showed both experimentally and theoretically that the duration of closure of the reed is independent of the played note, i.e. of the equivalent length of the resonator. This duration is related to the round trip of a wave over a length equal to this of the missing part of the cone, denoted in what follows x_1 (see Fig. 1).

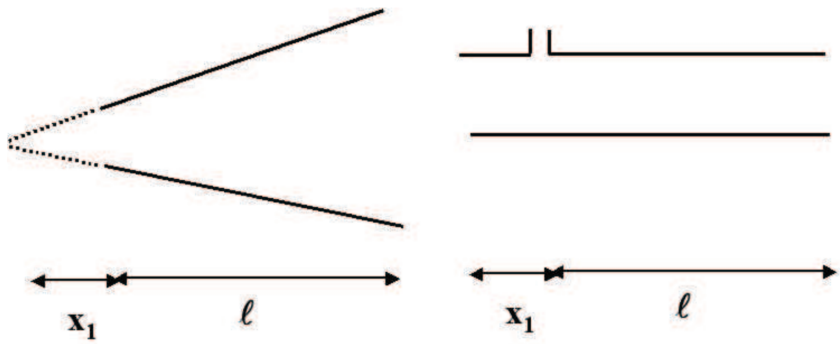

Figure 1. A truncated cone (on the left) and the equivalent "cylindrical saxophone'" (on the right). For the latter, the mouthpiece is placed on the side of the cylinder.

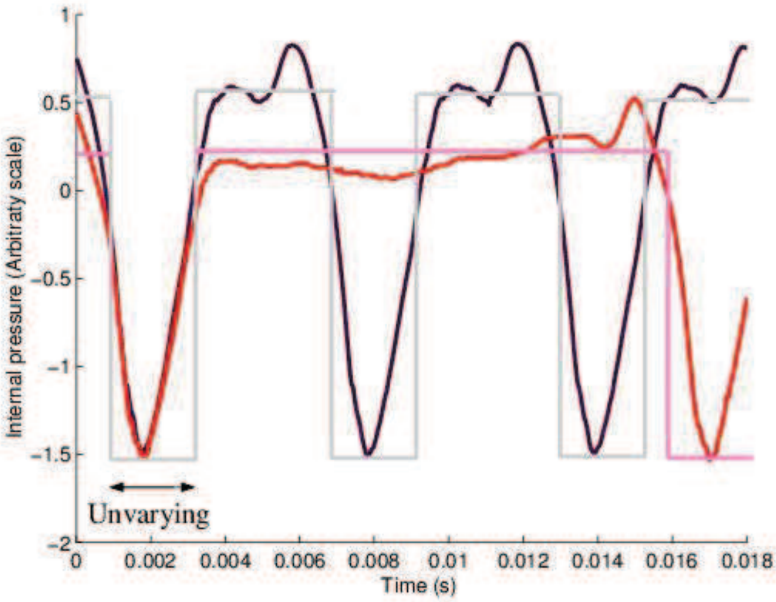

Figure 2. Periodic signal of barytone saxophone, and the approximation by a rectangle signal. The signal is the internal pressure for two notes, which are the lowest and the highest of the first register. The duration of the of negative pressure state, which corresponds to the reed beating, is common to the two notes.

This paper deals with this result, and its aim is to discuss some paradoxes and open questions. It is limited to the first register of these instruments, and simplifies the effect of toneholes in a limitation of the effective length of the truncated cone.

2. Analogy and paradoxes

Fig. 2 shows the measured signal of internal pressure for two notes of a baritone saxophone, corresponding to two values of the equivalent length ℓ. The measurement is done with a microphone inside the mouthpiece. If the shape is caricatured as a rectangle signal, the idea is to consider that the saxophone is equivalent to a bowed string instrument. The episode of negative pressure corresponds to the beating reed, and its duration does not depend on the played note, i.e. their equivalent length ℓ. The analogy between the sound pressure in the mouthpiece of conical reed instruments and the velocity of the violin string was proposed by Dalmont et al (2000), and Olivier et al (2004), showing that it should be possible to replace the conical instrument by what they called a "cylindrical saxophone" (see Fig.1), i.e. a cylinder of length $\ell + x_1$ open at the two ends excited at the distance x_1 from the left end. The idealized pressure signal looks like the well known Helmholtz motion, which is an idealization of the velocity of the violin bowed string.

The explanation of this analogy is based upon the assumption that the length of the missing part of the cone is small compared to the wavelength. Thus the input admittance of the truncated cone (without mouthpiece) can be approximated as:

$$Y = \frac{S_1}{\rho c}\left[\frac{1}{jkx_1} + \frac{1}{j\tan k\ell}\right] \quad (1) \quad \text{or} \quad Y \approx \frac{S_1}{\rho c}\left[\frac{1}{j\tan kx_1} + \frac{1}{j\tan k\ell}\right] \quad (2)$$

$k = \omega / c$ is the wavenumber, where $\omega = 2\pi f$ is the angular frequency, S_1 is the input cross section area of the tube, c the speed of sound and ρ the air density. We believe that the basis of this analogy was given first by Irons (1931), who concluded that the first resonance frequency is given by $f_1 = c/2L$ where $L = \ell + x_1$. This expression is a first approximation of the playing frequency. His explanation was completed by Benade (1959) and Nederveen (1969), with the result that the analogy is improved when the mouthpiece has a volume equal to that of the missing part of the truncated cone.

Similarly to what happens for the violin, the admittance given by Eq. (2) leads to several solutions for the self-sustained oscillation. One of these solutions is called the Helmholtz motion, which is a rectangle signal (when losses are ignored). This analogy leads to useful conclusion concerning important features of the sound production (regime of oscillation, amplitude), but accurate insight of the tone colour cannot be expected. A first evidence is the triangular shape during the closure state (see Fig. 2), differing from the rectangular shape of the pure Helmholtz motion. Nevertheless spectra of the internal pressure (i.e. the pressure in the mouthpiece) of saxophones exhibit anti-formants, at frequencies roughly corresponding to the harmonics of the fundamental frequency $c/2x_1$. On the one hand it is an argument in favour of the analogy with the Helmholtz motion, while on the other hand this result is paradoxical, because if the frequency corresponds to a round trip over length x_1, the wavelength is by definition of the order of magnitude of this length, thus the analogy fails. This will be discussed hereafter.

Because of the necessity to provide a mouthpiece, a perfect cylindrical saxophone is difficult to be built. A simple reasoning is the following: a truncated cone has a positive inharmonicity (because the series of resonances is intermediate between 1, 2, 3, etc and 1, 3, 5, etc.). As explained above, the mouthpiece partly corrects this inharmonicity by creating negative inharmonicity. Therefore for a cylindrical saxophone, the addition of the mouthpiece creates an inharmonicity whose sign is inverse of that of a conical instrument. However it was shown by Dalmont et al (2000) that a stepped cone with appropriate dimensions is equivalent to a perfect cylindrical saxophone, and with this device there is no problem with the mouthpiece. This instrument seems to be very similar to a true saxophone, when looking at the signal shape for the internal pressure. In particular, the episode of negative pressure has a similar triangular shape (and is not a constant, as for the pure Helmholtz motion). This is also paradoxical, because the input impedance is very close to that defined by Eq.

(2).
Recent numerical simulation of the sound production by a reed conical instrument (Kergomard et al 2012) showed that this triangular shape is obtained with the simplest model without losses and reed dynamic. Moreover it is obtained with a discontinuity of the derivative of the nonlinear function relating the pressure difference and the flow rate, i.e. with a brutal beating. Thus the triangular shape is not due to a smooth closure of the mouthpiece by the reed.

3. Theoretical results for the ideal case of a cylindrical saxophone: Internal and external pressure signals.

In this section, we consider the simplest approximation of a conical instrument, i.e. a perfect cylindrical saxophone, and the particular solution of the Helmholtz motion. We will obtain little information about the spectrum, because we ignore the losses. But some behaviour can be explained thanks to this simplified model. The great advantage is that the shape of the internal pressure signal is a priori known. We consider a periodic solution of period $T = 2L/c$. The pressure and the flow rate have the frequencies $f_n = nc/2L$ as *only possible components*, and they satisfy $\sin kL = 0$ (this can include the dc component).
The internal pressure signal $p(t)$ has the following spectrum:

$$P_n = -p_L sign(-p_L)^n \frac{\sin X_n}{X_n}$$
$$X_n = \frac{n\pi x_1}{\ell + x_1} = n\pi\beta = k_n x_1 = \frac{\omega_n x_1}{c} \qquad (3)$$

p_L is the pressure value during the longer episode and its value is related to the excitation parameters. $\beta = x_1/(\ell + x_1)$ is the ratio of the lengths on the two sides of the reed, using the classical notation for the bowed string. The flow rate $u(t)$ at the input is constant. As it is well known for a bowed string, if β is rational, the quantity $\sin X_n$ can vanish, and some harmonics are missing (the harmonics for which $n = m/\beta$, m and n being integers. If β is irrational, there are anti-formants for frequencies with $\sin X_n$ close to 0. The frequency of the nth harmonic is given by:

$$f_n = \frac{nc}{2(\ell + x_1)} = \frac{X_n c}{2\pi x_1}.$$

3.1 Transfer function from input to extremity ℓ

In order for the analogy to be consistent, the cylindrical saxophone is assumed to radiate by the end ℓ only. The output flow rate U_R, which is assumed to be a monopole source radiating in the surrounding space, is related to the input pressure by the following standard relationship valid for planar waves:

$$P(\omega) = \frac{\rho c}{S} j \sin k\ell \, U_R(\omega). \qquad (4)$$

In order to use the knowledge of the spectrum (Eq. (3)), the following equations can be written for the harmonic n:

$$\sin k_n\ell = \sin\frac{n\pi\ell}{\ell + x_1} = \sin[n\pi - X_n] = -(-1)^n \sin X_n \quad \text{and} \quad \cos k_n\ell = (-1)^n \cos X_n.$$

Therefore for the harmonic n Eq. (3) can be rewritten as:

$$P_n = -\frac{\rho c}{S} j(-1)^n \sin X_n \ U_{Rn}. \qquad (5)$$

3.2 External spectrum

Using Eqs. (3) and (5), except if $\sin X_n = 0$, the following result is obtained:

$$U_{Rn} = -j \frac{S}{\rho c} \frac{p_L}{X_n} \qquad (6)$$

For the case $\sin X_n = 0$, the solution is undetermined. This happens when $X_n = m\pi$ and $\beta = m/n$ is rational. For that case, it is possible to use the other transfer function, considering that the flow rate at the excitation point is divided into two parts, on the two sides of the reed:

$$U(\omega) = U_{x1}(\omega) + U_\ell(\omega) = -j \cot kx_1 \ P(\omega)\frac{S}{\rho c} + \cos k\ell \ U_R(\omega).$$

For a Helmholtz motion, the flow rate $u(t)$ is a constant, therefore for non-zero frequencies of the signal:

$$j \cot k_n x_1 \ P_n \frac{S}{\rho c} = \cos k_n \ell \ U_{Rn}$$

As a consequence, using Eq. (3):

$$-jp_L(-1)^n \frac{\cos X_n}{X_n} \frac{S}{\rho c} = (-1)^n \cos X_n U_{Rn}.$$

This leads again to the result (6). Therefore *the spectrum of the output flow rate at extremity ℓ is complete.* No harmonics are missing in the external spectrum; *neither formants nor anti-formants are expected,* in opposition to the internal spectrum. The signal is a saw-tooth signal, as noticed by Cremer (1984) for the analogous problem of the bowed string. Assuming a monopole radiation, the external pressure is proportional to the time derivative of the output flow rate (with a certain delay). Omitting the delay, the relationship between the external pressure at distance d and the output flow rate is the following:

$$P_{ext}(\omega) = j\omega_n \rho U_R(\omega) \frac{1}{4\pi d}.$$

Using Eq. (6), the spectrum of the radiated pressure is that of a Dirac comb:

$$P_{ext,n} = \frac{Sp_L}{4\pi x_1 d}.$$

3.3 The particular case of clarinet-like instruments

The previous result seems to be valid for the clarinet, a particular case of the cylindrical saxophone for which $\beta = 1/2$; but it is not. This requires some explanation. The clarinet is equivalent to a cylindrical saxophone of cross section $S/2$ (S being the cross section of the clarinet), of length 2ℓ excited at its middle and radiating *by the two ends*. It is easy to show that the flow rates U_{R1} and U_R at extremities x_1 and ℓ, respectively, satisfy the following relationship:

$$U_{R1,n} = -(-1)^n U_{R,n}.$$

Therefore because in practice the two extremities are at the same location, the sum of the flow rates is zero for the even harmonics (for the odd harmonics, it is twice this of one end). It can be noticed that more generally a true cylindrical saxophone, which would radiate by both ends, should have a complicated spectrum and directivity pattern, similar to that of a Boehm flute.

4. The spectrum of conical instruments

The question of the relation between the conical shape and the sound spectrum remains largely open, in particular for the radiated sound. As explained previously, the hypothesis that it is equivalent to a cylindrical saxophone has a limited frequency range, because the length of the missing cone, denoted x_1, is assumed to be small compared to the wavelength. We have seen that the pressure signal during the episode of reed beating is common to the different notes, and we can conjecture that characteristic frequencies common to different notes exist in the spectrum, including higher frequencies, and a consequence is the apparition of formants or anti-formants. For the lowest ones, the common frequencies are close to those of a cylindrical saxophone.

4.1 Internal spectrum

Let us consider first the spectrum of the internal pressure at low frequencies. We can imagine without rigorous proof that the maximum of this spectrum is linked to that of the input impedance. The latter can be determined by using a formula which gives the envelope curve of the impedance peaks for a truncated cone (Chaigne 2008). The envelope is proportional for all notes of the following curve:

$$\frac{1}{1+\frac{1}{k^2x_1^2}}\frac{1}{\sqrt{kx_1}}.$$

A simple calculation gives a rough estimation for the position of this maximum: $kx_1 = \sqrt{3}.$ A formant can be found around the corresponding frequency. For instance, for a soprano saxophone, it is 670 Hz. This approach should be confirmed by a more complete analysis. For the lowest notes of an instrument, there are few harmonics 1 or 2 in the spectrum because of the shape of the input impedance curve. This is particularly true for a bassoon.

What happens at higher frequencies? Obviously if the quantity kx_1 is of order of π or larger, the assimilation to a cylindrical saxophone has no longer any meaning. Nevertheless it is possible to find frequencies which are independent of the notes and for which the input impedance is minimum (and also frequencies for which the input impedance is maximum). We first forget the losses. As we have seen, the frequency of the first maximum at the input of the mouthpiece is, an excellent approximation apart, given by $c/2(\ell+x_1)$, thus $k(\ell+x_1)=\pi$: we know that it is not exactly the playing frequency, because the latter depends on the excitation level, but we assume that it is true. The harmonics of this frequency are not necessarily resonance frequencies, because they do not satisfy the condition $kx_1 << 1$. However the harmonic n satisfies:

$$k(\ell+x_1)=n\pi \quad \text{thus} \quad \cot k\ell = \cot(n\pi - kx_1) = -\cot kx_1.$$

In other words, for a given frequency which exists in the spectrum, the input admittance of the truncated cone (and therefore the admittance projected to the input of the mouthpiece) does not depend on the fingering, i.e. on the played note. These admittances exhibit extrema which are common to the different notes, as shown in Fig. 3 (the calculation shown includes losses). Thus in the spectrum of the internal pressure formants and anti-formants are expected: they are the elements common to the different notes that we above mentioned. They can be calculated for a given shape of the mouthpiece, but here we do not discuss this matter further. The first anti-formant frequency is slightly higher than this of a cylindrical saxophone ($kx_1 = \pi$). But these formants and anti-formants are less accentuated than those of a cylindrical saxophone, for two reasons: i) because losses make their existence less evident; ii) the harmonics of certain notes are close to such a frequency, but this is wrong for other notes, for which these harmonics are less attenuated.

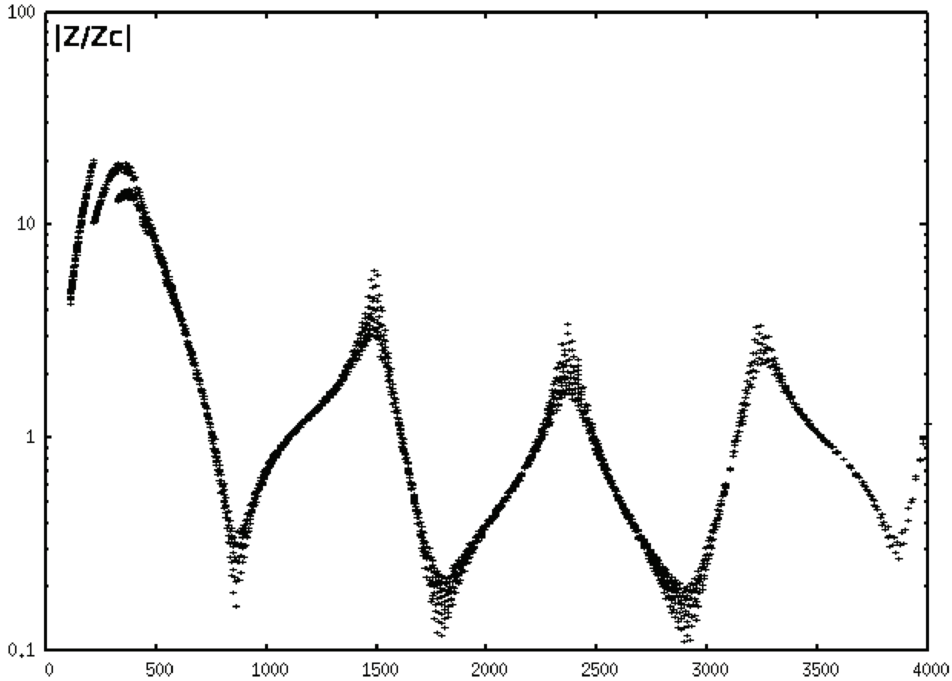

Figure 3. Input impedance for 100 values of ℓ linearly distributed on one octave. The visco-thermal losses are taken into account. The points indicate the impedance modulus for the fundamental frequency and its harmonics for the notes corresponding to each value of the length. The extrema are noticeable, they are common to the different notes. The dimensions are those of a tenor saxophone with a mouthpiece. The toneholes are ignored (the change of note is given by a modification of ℓ)

4.2 External spectrum

The previous analysis is concerned by the internal spectrum. What happens for the radiated pressure? For the lowest frequencies the previous reasoning can be extended by using directly the shape of the input impedance curve. The relative weakness of the lowest frequencies with respect to that of the internal pressure is accentuated by the fact that the radiated pressure is the time derivative of the output flow rate.

Unfortunately for the higher frequencies we have no simple insight. Formants and anti-formants seem to exist, but they are less evident and their position differs from that observed for the internal pressure. For a cylindrical, clarinet-like instrument the issue of the relative amplitude of the even and odd harmonics is subtle. We imagine that this subtlety is similar for a cylindrical saxophone for the main harmonics versus the missing ones in the internal pressure. The problem becomes more intricate for a conical instrument (remind that this discussion ignores the existence of toneholes, which strongly complicate the sound analysis above the cutoff frequency, and moreover the reed dynamic is ignored.

What is clear is that the level difference between formants and anti-formants is much smaller than for the internal pressure, similarly to the difference between even and odd harmonics of the clarinet. This is due to the difference between the input impedance and the pressure transfer function: the first function of the frequency has poles and zeros, while the second has poles only. Nevertheless, contrary to some possible hypotheses, the position of the formants is neither directly linked to the length x_1 nor to the mouthpiece shape. A consequence can be deduced from the previous analysis: if the length x_1 of the missing cone is reduced, and then if for a given length, the apex angle is increased, the first characteristic frequencies increase. Observing the increase of the taper from the first saxophones of Adolphe Sax to modern saxophones, it was possible to explain this increase by the aim to enrich the timbre (see Kergomard (1998)), as probably requested by the jazz music. Comparing with a violin, this would correspond to a play closer to the bridge: it is well known that the timbre is richer in harmonics because the first missing harmonic becomes higher (similarly to what happens for a guitar, producing free oscillations). Finally we mention the study by Benade and Lutgen (1988), who measured the external spectrum averaged in a room and showed for the highest frequencies the existence of a frequency above which the spectrum decreases as f^3: after these authors, this frequency would be linked to the cutoff frequency of the toneholes lattice. Moreover they showed that minima exist in the spectrum of a given note, and that is related to the reed beating.

Perspectives

To our mind many issues remain to deepen about this matter. Some works are in progress concerning the use of the simplest model in order to understand the effect of the mouthpiece. Experiments with an artificial mouth will be necessary, starting with tubes of different lengths, radiating by one orifice only.

Acknowledgments

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